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No. 5.

PAPERS
ON
PRACTICAL ENGINEERING.

PUBLISHED BY THE ENGINEER DEPARTMENT,

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OFFICERS OF THE UNITED STATES CORPS OF ENGINEERS.

O

PAPERS ON PRACTICAL ENGINEERING.

No. 5.

AN ANALYTICAL INVESTIGATION OF THE RESISTANCE OF PILES TO SUPERINCUMBENT PRESSURE,

DEDUCED FROM THE FORCE OF DRIVING; WITH AN APPLICATION OF THE FORMULA TO THE

FOUNDATIONS OF FORT MONTGOMERY,

Rouse's Point, N. Y.

BY

BREVET LIEUT. COLONEL JAMES L. MASON,
CAPTAIN U. S. ENGINEERS.

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BVT. BRIG. GEN. JOS. G. TOTTEN,
CHIEF ENGINEER, *Washington, D. C.*

ROUSE'S POINT, CLINTON CO., N. Y.,
January 12*th*, 1850.

SIR: I submit the following remarks on the timber foundation (of piles and grillage) of Fort Montgomery.

Fort Montgomery is founded on a timber platform resting on 4383 piles driven into the ground. These piles were driven in 1844, 1845 and 1846, and the grillage laid on them in those years and in 1848. These piles, when the work is completed, will bear an average load of seven cubic yards of masonry, and one and three fourths cubic yards of earth each; or, estimating the masonry at 4200 lbs., and the earth at 2700 lbs., per cubic yard, the weight with which each pile will be loaded, will amount to 34125 pounds, in addition to its own weight and that of the grillage.

These piles were all driven by two steam engines, one of six horse, and the other of eight horse, power; the former requiring generally an engineer and three laborers to serve it, and the latter an engineer and four laborers.

The original cost of engines was $4388.71;—the consumption of oil and rope, together with the repairs applied to them, has amounted to $1982.21;—making the total expended on the engines $6370.92: I suppose they could now be sold for $1000, a sum not over one-third of their value as pile drivers, if they were further needed at this work for that purpose: deduct the $1000, and it leaves the amount actually bestowed on the pile driving

machinery for 4383 piles,	$5370.92	or	$1.22	pr. pile.
The cost of the piles was -	6121.42	"	1.40	"
The cost of driving was -	1724.44	"	40	"
The cost of measuring, hauling, securing for winter, and sharpening, was -	792.24	"	18	"
The cost of iron bands, to protect the head of the piles while driving, was -	439.29	"	10	"
The cost of cutting down and levelling the piles with axe and adze to receive the grillage, was -	495.21	"	11	"
The machinery other than steam pile drivers, - -	184.10	"	04	"
The cost of contingent services and contingencies for this part of the work amounts to - - -	1890.05	"	43	"
Making the total cost of piles and driving, - -	$17017.67	or	$3.88	pr. pile.

The general arrangement of the piles aimed at, was to place their centres under the four angles of a square yard; the double pile driver having been so constructed as to drive two rows two yards apart from centre to centre, and on repassing the same track to insert an intermediate one together with one outside the two already driven; but the distance from pile to pile in each row, resulting generally from dividing a given length into a certain number of spaces, would vary an inch or two from three feet.

The grillage was laid in two courses; the lower, of timber 1′ 3″ wide and one foot thick, was generally laid perpendicular to the scarp, thus connecting together rows of piles parallel to the scarp. It was notched down four inches on to the piles and pinned. The upper course, at right angles to the lower, was of 12″×12″ timber, across the piles, and of 12″×8″ between them; the 12″×12″ being notched 4″ down on the lower course, brought its top to the same level as the 12″×8″—thus giving a level floor for the masonry. This arrangement is shown on the sketch herewith.

The materials for the grillage, consisting of 45610 running feet of timber of the sizes, 12″×8″, 12″×12″ and 12″×15″, and of 12147 hard wood pins cost,	$2277.45
The cost of measuring, hauling and securing this timber for the winter, was	235.80
The cost of preparing, laying and pinning it,	2318.02
The cost of machinery,	70.00
The cost of contingent services and contingencies belonging to this part of the work, was	612.66
Making the total cost of the grillage,	$5513.93

The difference of level between the highest and lowest water in this part of Lake Champlain, is nearly 8 feet: according to our memoranda 7′ 10″. To prevent the decay of the wood, it was necessary to place the top of the grillage at least as low as the lowest level of the surface of the water;—the piles had to be accurately levelled 1′ 4″ below the top of the grillage. It thus became necessary, let the stage of water be what it would, to enclose the area with a coffer dam and dyke and to pump out the water.

The cost of the coffer dam and dyke, together 1700 feet long, and enclosing an area of $2\frac{3}{4}$ acres, was	$1395.93
The cost of pumping,	1504.21
The cost of excavating for the platform for the piles and grillage,	1587.87
Contingent services and contingencies belonging to the last three items,	561.00
Total cost,	$5049.01

We have then—

For excavating, pumping and coffer dam,	$5049.01
For grillage,	5513.93
For piles,	17017.67
The sum total of expenses of every description, incurred in preparing the timber foundation of the fort—including contingent services and contingencies of all sorts,	$27580.61

To investigate the value of this foundation, or the amount of weight with which each pile may be loaded, let h be the height of the fall of the hammer at the last blow; $g=32\frac{1}{6}$ feet, the force of gravity, or the velocity that gravity will generate in a second of time; then, omitting the inconsiderable resistance from the friction of the hammer against the slides, $\sqrt{2gh}$ will be the velocity of the hammer at the moment it strikes the head of the pile.

Calling this velocity V, and H the *mass* of the hammer, the quantity of motion at the instant of impact will be VH, and the hammer and pile will move together.

Calling V' the velocity of the joint mass of hammer and pile at the instant of impact, and P the *mass* of the pile, the quantity of motion will be $V'(H+P)=VH$, or

$$V'=\frac{VH}{H+P} \qquad (1)$$

for the velocity of the pile at the instant the retarding force, due to the difficulty of penetrating the soil, begins to act.

But the *mass* of the pile or hammer is equal to its weight divided by g; or calling the *weights* h' and p we have, $H=\frac{h'}{g}$ and $P=\frac{p}{g}$; substituting these values in expression (1) and reducing by multiplying both numerator and denominator by g, we have

$$V'=\frac{Vh'}{h'+p} \qquad (2).$$

Let the joint *mass* $(H+P)$ of the pile and hammer, be now taken as the unit of mass, and the variable t, the time that it is in motion under the influence of the retarding force;—let φ represent this force, whether constant or variable. Let the variable v represent the velocity destroyed by the retarding force, and we shall have

$$\varphi=\frac{dv}{dt}, \qquad \text{or} \qquad dv=\varphi.dt \qquad \text{and} \qquad v=\int^{t}\varphi.dt$$

if the integral be taken to the end of the time during which the pile is in motion, v, the velocity destroyed, becomes V' and we have

$$V'=\int\varphi.dt \qquad (3).$$

The velocity at any moment will be $V'-v$; let s be the distance to which the pile has penetrated at that moment, and we have $\frac{ds}{dt} = V' - v$: (the differential of the space divided by that of the time being equal to the velocity:) integrating we have

$$s = V't - \int v.dt \quad (4) \qquad \text{but} \qquad v = \int \varphi.dt;$$

substituting this value of v in equation (4) we have

$$s = V't - \int dt. \int \varphi.dt. \quad (5).$$

To integrate equations (3) and (5) it is necessary to know the form of the expression φ; it may be constant, or variable, with the time t. Let us for a moment, suppose it constant. Equation (3) becomes $V' = \varphi t$. (7) and equation (5), $s = V't - \int \varphi.t.dt$ or

$$s = V't - \tfrac{1}{2}\varphi.t^2. \quad (8)$$

Substituting in the latter the value of $\frac{1}{2}\varphi.t^2$, deduced from equation (7) in which t represents the *total time* to *the end* of the motion, we have $s = V't - \frac{1}{2}V't = \frac{1}{2}V't$, or

$$t = \frac{2s}{V'} \quad (9).$$

Substituting this value of t in equation (7) we have

$$\varphi = \frac{V'^2}{2s} \quad (10)$$

for the retarding force on the unit of mass (the joint mass of pile and hammer.)

Squaring equation (1) we have $V'^2 = \frac{V^2 H^2}{(H+P)^2}$; substituting for V^2 its value $2gh$, $V'^2 = \frac{H^2}{(H+P)^2} \times 2gh$; substituting this value of V'^2 in expression (10) it becomes,

$$\varphi = \frac{H^2}{(H+P)^2} \times \frac{gh}{s};$$

multiplying both numerator and denominator by g, we have,

$$\varphi=\frac{H^2g^2}{(H+P)^2g}\times\frac{h}{s}$$

for the value of the retarding force acting on the assumed unit of mass; now to pass from this unit to whatever unit H and P may be expressed in (as cubic inch, cubic foot, cubic yard, of a particular density) we will multiply by $(H+P)$ and we have for the total retarding force

$$(H+P)\varphi=\frac{H^2g^2}{(H+P)g}\times\frac{h}{s}. \qquad (0)$$

But the mass multiplied by gravity gives the weight; the first factor, therefore, is the product of the weight of the hammer, by the ratio of that weight to the joint weight of pile and hammer, and this must be multiplied by the ratio of the height of the fall to the distance that the pile is driven, to give the value of the retarding force in terms of the weight that it will hold in equilibrio.*

**Note*—The above expression (0) gives properly speaking that portion of the retarding force which is employed in destroying the motion resulting from the *blow* of the hammer. The total retarding force must not only do this, but must in addition, neutralize the action of gravity on the mass of pile and hammer during the operation. So much of the intensity of the retarding force, as is necessary to balance the weight of the pile, is always in requisition for that purpose, and entirely unavailable for the support of the superstructure with which the pile is loaded. To the above expression for the admissible load, the weight of the hammer might be added, but this is inconsiderable.

If the effect of gravity on the pile and hammer be introduced into the foregoing calculations, equation (10) will become

$$\varphi-g=\frac{V'^2}{2s} \qquad (10')$$

and equation (9) will remain as it is,

$$t=\frac{2s}{V'} \quad \text{or} \quad s=\tfrac{1}{2}V't, \qquad (9').$$

Now if the hammer be merely laid on the pile, $h=0$, $V'=0$, and from equation (9') $s=0$, equation (10') then becomes

$$\varphi=g+\tfrac{0}{0}.$$

Herewith are two sketches, the first giving the history of the driving of the piles, the second showing how the grillage was laid on them, in the only portion of the work where the entire load has been placed. It is the only part of the work where the parapet wall surmounting the scarp, has been built.

In the first sketch, the diameters of the butt and point (in inches) are written within the circle representing the pile—the red figure within the red curve, gives the length after it was cut off to receive the grillage, the red figure, without that curve, gives the length before it was cut off;—the black figure without the red curve gives the fall of the hammer at the last blow, and the series of black figures within that curve, are, in inches, the penetrations of the pile into the soil under the corresponding final blows, the lower figure belonging to the last blow:—the wood of the pile is represented by the letters, *A* for ash, *B* for beech, *H* for hemlock, *S* for spruce and *T* for tamarack.*

The hammer with which these piles were driven, weighed 1630 pounds. The weight of the spruce piles was found to be $39\frac{3}{5}$ pounds to a cubic foot, but the piece weighed was rather more dry than the average of the

---

This is as it should be; the force has given no measure of its energy, such as the space or time in which it can destroy a given quantity of motion; it has borne the weight of the pile and hammer; how much more it could have borne is uncertain, and the expression takes the indeterminate from of $\frac{0}{0}$. Making $\varphi = g$ in equation (10′) s must be infinite; if φ be supposed less than g, then $\frac{V'^2}{2s}$ becomes negative, and V' (the square root of a negative quantity) imaginary. In this last case V'^2 may be made positive, and V' become real, by making s negative, which is equivalent to the supposition that the blow of the hammer is on the bottom of the pile, and drives it upwards.

* Only one of the sketches referred to, is published. The information contained in the above paragraph, will be found in a table at the end of the article.

piles;—they may be assumed to have averaged at least 40 pounds to a cubic foot.

Now taking the central pile of the front row of the scarp, in the sketch, its volume is 24 cubic feet, its weight 960 pounds; its joint weight with hammer 2590:

$$\frac{h}{s} \text{ is } \frac{36'}{3\frac{3}{4}''} = 115\tfrac{1}{5},$$

and we have, for the value of the retarding force

$$1630 \times \tfrac{1630}{2590} \times 115\tfrac{1}{5} = 118175 \text{ pounds.}$$

This is about an average result of the calculation applied to the others.

Now each pile, in the sketch, supports the average weight of six cubic yards of masonry and one and a fourth of earth; or taking 4200 pounds as the mean weight per cubic yard of our masonry, and 2700 pounds as that of the earth, the load on each pile amounts to 28575 pounds, besides its own weight and that of the grillage; about one fouth of that which the force of resistance is capable of holding in equilibrio. These piles have been loaded with 23800 pounds since the fall of 1846, (for three years,) and the addition to that load this season, makes the total per pile, 28575.

In the case just calculated, the velocity of the hammer at the moment of impact will be $\sqrt{2 \times 36 \times 32\frac{1}{6}} = 48.125$ feet per second, and the velocity of the joint mass of hammer and pile will be $48.125 \times \frac{1630}{2590} = 30.3$ feet per second $= V'$, and the retarding force $\frac{V'^2}{2s}$ will be

$$\frac{(30'.3)^2}{7\frac{1}{2}''} = \frac{918'}{7\frac{1}{2}''} = 1468.$$

The velocity that the retarding force is capable of destroying in a second of time on the joint *mass* of the pile and hammer is then 1468 feet—but gravity is capable of generating on that mass a velocity of $32\frac{1}{6}$ feet; dividing 1468 by $32\frac{1}{6}$, we have $45\frac{63}{100}$, as the ratio between the retarding force and the force of gravity: that is, the retarding force is capable of holding in equilibrio $45\frac{63}{100}$ times the joint *weight* of the pile and hammer or 2590 pounds $\times 45\frac{63}{100} = 118181$ pounds; within 6 pounds of the result given above, and this difference would not have appeared if the calculations had been extended to a greater number of places of decimals.

The time t in this case will be

$$t = \frac{2s}{V'} = \frac{7\frac{1}{2}''}{30'.3} = \frac{2}{97} \text{ of a second.}$$

The same result may be obtained by dividing the velocity destroyed, viz: 30′.3, by the velocity the force is capable of destroying in a second of time, viz: 1468; or $\frac{30.3}{1468} = \frac{2}{97}$.

Bastion A, the bastion nearest the channel, was finished in the fall of 1846 with the exception of the parapet wall surmounting the scarp. In that bastion, the piles were longer and heavier than those in the sketch herewith; they also went further under the last blows. The above calculation applied to those in the neighborhood of the salient would give about 85000 pounds as the load they could sustain. They have sustained for three years 23700 pounds each; the parapet wall will add about 4000 pounds to each pile.

The above calculation has been made very easy, by the supposition that the retarding force is constant during the very short period that it takes to destroy the motion of the pile. In the case of the pile taken above as an example, the calculation gives $\frac{2}{97}$ of a second on the supposition of a constant force. This time was not measured. To measure it, might be a difficult mechanical problem. Still I think an apparatus could be devised that would effect it.

Suppose this time had been measured, and had not agreed with that given by equation (9): it would show that the assumption of a constant force was not correct. Now the measured time might be either greater or less than $\frac{2s}{V'}$; if less, then the average intensity of the variable force must be greater than that of the constant force, and its greatest intensity must, *a fortiori*, be so.—If however, the measured time should be found greater than $\frac{2s}{V'}$, the variable force must still, at some moment of its progress, earlier than the lapse of time $\frac{2s}{V'}$, have become greater than the constant force $\frac{V'^2}{2s}$: for if it were at every moment less, its effect at every moment to reduce the velocity of the pile must have been less, and the pile would at every moment move with a greater velocity than it would under the retarding influence of the constant force $\frac{V'^2}{2s}$, and of course at the expiration of the time $\frac{2s}{V'}$ it would have penetrated a greater distance than s. Thus on the supposition of a variable force, let the total time that it takes to arrest the motion of the pile be either greater or less than that taken by the constant force, yet it must, at some moment, have a greater intensity than the constant force.

Now the variation of the force may be according to a simple or a complex law. Its variations may be steadily in one direction, that is constantly *(not uniformly)* to increase or constantly to diminish the intensity that the force had at the beginning of the motion—or they may be the reverse, tending during one portion of the time to increase and at another to diminish the value of the force at the commencement of the motion. Any law of variation that does not look to the continual increase or continual diminution of the force from the commencement, seems to me highly improbable.

If it be assumed that the variations of force are steadily in the same direction, that is continually increasing or continually diminishing it, ***its greatest intensity*** must be at either the commencement or the close of the motion of the pile. If at the close of the motion, it will be the resistance that the pressure, if at once applied, would have to overcome to start the pile; and if at the commencement, it will be greater than the constant force determined by the data given by the next blow. What penetration (s), another blow would have given, cannot be said with certainty; but as the piles were always driven until for several successive blows, there was a steady diminution of (s), there is every probability that the constant force determined by another blow, would have been about the same as determined by the last one and perhaps a little greater.

Thus it is certain that if the force be variable, it must at certain moments be greater than the constant force determined by equation (0); and that unless the variations are governed by some complex and improbable law, that among those moments must be the commencement or the termination of the motion.

Then the assumption of no variation in the intensity of the force of resistance is either true, or it will give values less than the true ones and is therefore a safe supposition.

There might be circumstances perhaps, to make the intensity of the retarding force greater at some intermediate moment than at the commencement or close of the motion; as, for instance, *a very thin* hard stratum just below the point of the pile; before the pile should reach it, the penetration would be easy; it would be difficult while passing through it, and again become easy. But there were no such circumstances in the cases spoken of above. The borings in Bastion *A* down to the depth of 35 feet gave a mixture of very fine clay and sand, so fine that when dried and rubbed between the fingers it made an almost impalpable powder.

The velocity of pile and hammer destroyed, gives us one equation (3). The space moved over while this destruction of velocity is being effected gives another (5). These equations are sufficient to determine two unknown quantities and no more. The time not having been measured constitutes one unknown quantity. If then we assume an expression for a variable force that shall contain not more than one unknown co-efficient, those equations will suffice to determine both it and the time. If the time had been measured, the equations would have been sufficient to determine the variable force if it contained two unknown co-efficients.

Let us now suppose that the retarding force, having a certain intensity at the commencement of the motion, either increases or diminishes in proportion to the time t; we will begin with the case in which it diminishes, and assume the expression $\varphi = A\,(1-t)$ in which A is an unknown constant:—substituting this value in equation (5), we have

$$s = V't - \int dt.\int A\,(1-t)\,dt = V't - \int dt\,(At - \tfrac{1}{2}At^2)$$
$$= V't - (\tfrac{1}{2}At^2 - \tfrac{1}{6}At^3).$$

Substituting the above value of φ in equation (3), we have,

$$V' = \int A\,(1-t)\,dt = At - \tfrac{1}{2}At^2. \qquad (11)$$

Substituting this value of V' in the above expression for s, we have,

$$s = At^2 - \tfrac{1}{2}At^3 - (\tfrac{1}{2}At^2 - \tfrac{1}{6}At^3),$$

or, reducing,

$$s = \tfrac{1}{2}At^2 - \tfrac{1}{3}At^3. \qquad (12.)$$

Dividing both members of equation (12) by those of equation (11) we have,

$$\frac{s}{V'} = \frac{\frac{1}{2}t^2 - \frac{1}{3}t^3}{t - \frac{1}{2}t^2} = \frac{3t - 2t^2}{6 - 3t}:$$

or $$6s - 3ts = 3tV' - 2t^2V',$$

or $$2V't^2 - 3V't - 3st = -6s.$$

Consequently, $$t^2 - \tfrac{3}{2}(1 + \tfrac{s}{V'})\,t = -3\tfrac{s}{V'}:$$

This is a quadratic equation of the form $x^2 - px = -q$, in which the roots are

$$x = \tfrac{p}{2} \pm \sqrt{\left(\tfrac{p}{2}\right)^2 - q}:$$

comparing this with the above, we have,

$$t = \tfrac{3}{4}\left(1 + \tfrac{s}{V'}\right) \pm \sqrt{\left[\tfrac{3}{4}\left(1 + \tfrac{s}{V'}\right)\right]^2 - 3\tfrac{s}{V'}}$$

Of these two values of t, it is evident that the one in which the radical is affected with a negative sign, is the one belonging to the case; for $\frac{s}{V'}$ being always very small, the positive sign of the radical would give t greater than unity; but φ, the retarding force, being equal to $A\,(1-t)$, becomes negative when t is greater than unity and gives an accelerating instead of a retarding force.

To apply this expression to the case of the pile already referred to, in which we had $V' = 30'3$, and $s = 3\frac{3}{4}'' = '.312$, we have,
$\frac{s}{V'} = .010314$, $\left[\frac{3}{4}\left(1+\frac{s}{V'}\right)\right]^2 = .574163$, $.574163 - 3\frac{s}{V'} = .543221$, $\sqrt{.543221} = .737035$, $\frac{3}{4}\left(1+\frac{s}{V'}\right) = .757735$ and the difference $t = .0207$ of a second.

From equation (11) we have $\mathcal{A} = \frac{V'}{t - \frac{1}{2}t^2}$; but $V' = 30'.3$ and $t = .0207$; applying these values we have $\mathcal{A} = 1479$; But $\varphi = \mathcal{A}(1 - t)$: At the commencement of the motion, $\varphi = 1479$, and at the close, $\varphi = 1479\,(1 - .0207) = 1448\frac{1}{2}$.

Thus in the case in which the supposition of a constant force gave $\frac{2}{97}$ of a second for the time, and 1468 feet as the velocity per second that the retarding force is capable of destroying on the mass of hammer and pile, the supposition of a force *varying* according to the above law, gives .0207 of a second for the time, and 1479, and $1448\frac{1}{2}$ as the velocity per second that it can destroy at the commencement and close of the motion. These results do not vary much from those of the supposition of a constant force, but showing the force to be stronger at the commencement of the motion they tend to confirm the reasoning above as to the safety of that supposition.

If we take the case of the force increasing as the time, and assume $\varphi = \mathcal{A}(1 + t)$, the solution of the equations (3) and (5) with this value of φ will give

$$t = -\frac{3}{4}\left(1 - \frac{s}{V'}\right) \pm \sqrt{\left[\frac{3}{4}\left(1 - \frac{s}{V'}\right)\right]^2 + 3\frac{s}{V'}}$$

and

$$\mathcal{A} = \frac{V'}{t + \frac{1}{2}t^2}$$

The upper sign of the radical must be adopted, for the lower one would always give t negative. Applying these expressions to the case of the pile so often referred to, we shall find $t = \cdot 02056$ of a second, and $\varphi = 1459$ at the beginning and $\varphi = 1489$ at the close of the motion; one of these results being less and the other greater than that of the constant force.

Towards the salient of Bastion *A*, the piles ought to be able to endure a weight of 85000 pounds; they have been loaded for three years with 23700 pounds and there has been no motion. Thus Fort Montgomery gives three years experience in favor of a co-efficient of stability of $3\frac{6}{10}$ when applied to the calculation under the supposition of a constant force.

Many cases occurred in which, after a pile had been driven some days, another blow was struck, and the result was *invariably* a less motion than was to have been expected, if this blow had immediately succeeded the others. One of these cases occurred in the sketch herewith, in which a pile, that on the 18th of September was driven from $4\frac{1}{4}$ to $5\frac{3}{4}$ inches under the last four blows, on being struck again on the 20th September was driven but 2 inches.

In stating the average weight with which the piles are loaded, no deduction has been made for the influence of the water (in the spring months) in reducing the weight of the masonry. In certain seasons and for a short time this might reduce the loads from four to five per cent.

I am very respectfully,
Your obedient servant,
J. L. MASON,
Captain Engineers Bvt. Lt. Colonel.

TABLE showing the circumstances under which the several piles shown in the plate were driven.

No. of Pile.	Kind of Wood.	Diameter of pile in inches.		Length of pile before cutting off.		Length of pile after cutting off.		Fall of Hammer at the last blow.		Several penetrations of pile with a few of the last blows in corresponding order.
		Butt.	Point.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	INCHES.
1	Spruce.	12½	9¾	32	11	32	0	34	10	5¼-4¼-3⅞-3⅝-3½.
2	do.	17½	13	33	0	31	2	33	11	4¼-3⅝-3⅛-2¾-2⅜.
3	do.	14	11½	32	0	31	4	35	1	10-6⅞-5½-4⅝-4⅛-3⅝.
4	do.	14	9¾	33	0	31	8	34	5	5-4½-4-3½.
5	do.	13¾	10¼	32	0	32	0	36	0	5¼-4¾-4⅜-4⅛-3¾.
6	do.	14¼	10¼	32	0	32	0	35	10	5-4⅝-4⅝-4⅛-3⅜.
7	do.	14¾	10	32	0	31	10	35	7	4¾-4¾-4½-4¼.
8	do.	14¾	10	33	0	32	1	34	10	5¼-4½-4⅛-4-3⅝.
9	do.	14¼	10½	32	0	31	5	35	3	5⅜-5-5-4½.
10	do.	14	11	33	0	32	9	35	8	4-3½-3¼-2⅞-2½.
11	do.	12	9½	33	0	33	0	35	9	5-4¼-3¾-3½-3⅜-3⅛-3-3.
12	Hemlock.	14	11	29	11	29	2	35	1	11-10¼-9½-9¼-7¾-6½.
13	Spruce.	13¾	9	31	0	30	2	35	1	10-8¾-6⅜-5-4½-3⅞.
14	do.	14½	12	31	3	30	2	34	11	6-4¾-3¾-3½-3.
15	do.	12¾	9¼	31	4	31	0	35	8	9¼-6½-5-4¼-3¾-3¾.
16	do.	13¼	9¾	30	9	30	6	35	9	6-5-4½-4-4¾.
17	Tamarack.	15	11½	29	11	29	2	35	1	8-7-6-4¾-4¼.

18	Spruce	15½	11¾	32	1	28	7	32	6	5-4-3¾-3½-3¼-3-2¾.
19	do.	16	14¾	31	9	28	1	32	1	5-3¾-3¼-3-3-3-3¼.
20	do.	12¾	9¼	31	6	31	1	35	3	17-11-7¾-6¼-5-5¼.
21	do.	13	9¾	31	0	30	6	35	3	29-27-16-9¾-6½-5.
22	do.	14½	10¾	32	0	29	2	33	3	9½-7-5¼-3¾-3¼-2¾.
23	Hemlock	15¼	10½	31	10	29	10	34	1	6⅛-5¼-4¼-3⅞-3⅜.
24	Spruce	16	11	31	4	29	11	34	5	5⅜-4¼-3¾-3⅜-3⅛.
25	do.	13	10	32	5	30	5	33	10	7¾-6¼-5⅛-4½-3¾.
26	do.	14¼	10¼	32	0	29	9	33	8	5-4½-4-3¾.
27	do.	14	10¼	33	0	31	4	34	5	5½:4½-4¼-4-3¾.
28	do.	13½	9	33	0	31	4	34	4	5¼-4¾-4½-4-3¾.
29	do.	15	9½	33	0	32	6	35	5	3⅜-3¼-3-3-3-3.
30	do.	16½	10	30	10	30	7	35	8	9½-7¾-5¾-4¾-3½-3⅛.
31	do.	13	9	33	0	32	3	35	3	4½-4¼-3¾-3⅝.
32	do.	13	11	32	0	29	8	33	11	5½-4½-3¼-3¼-3¼.
33	do.	13	10	30	2	29	9	35	7	11¼-9-7½-5.
34	do.	15	10	32	0	28	9	33	4	7-7¼-5½-5¼-4¾-5-4½.
35	do.	13	9½	30	4	28	8	34	6	5-5-5-4½-5.
36	do.	12	9¼	30	9	27	0	32	8	10¼-7-5¼-3¾-3½-
37	do.	14½	10½	29	7	28	4	35	0	4¼-4⅛-3⅝-3½-3¼.
38	do.	14½	10¼	34	0	29	10	31	7	7¼-5¼-4¾-4.
39	Beech.	13¾	10¼	32	7	31	3	34	6	4½-4-3½-3½-3½.

TABLE showing the circumstances under which the several piles shown in the plate were driven.

No. of Pile.	Kind of Wood.	Diameter of pile in inches.		Length of pile before cutting off.		Length of pile after cutting off.		Fall of Hammer at the last blow.		Several penetrations of pile with a few of the last blows in corresponding order.
		Butt.	Point.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	INCHES.
40	Tamarack	15	11	32	0	27	3	31	1	4½-3½-3-3⅜-3⅛.
41	Spruce	12	9	30	6	30	3	35	8	20-19-6½-5¼-4¼.
42	do.	13¾	9¼	31	4	30	1	34	7	4¾-4⅝-4¾-4⅝-4-3¼.
43	Hemlock	12½	10	30	10	28	5	34	0	6-4¾-4¼-4-4¼-4-4.
44	Spruce.	13	10	30	8	26	8	31	11	12¼-12¾-18½ broke bel.-rotten
45	do.	13	9	30	9	30	0	35	2	12¾-10¼-8½-7½-6½.
46	do.	12½	9½	32	6	31	10	35	3	6½-6½-6¼-6⅜-6⅛-6½-6-5¼.
47	do.	12	9½	32	0	31	5	35	4	6½-3⅞-5-5½-6-6⅞-9-7-6.
48	do.	12	10¾	30	7	30	7	35	10	5½-5¾-5¼-5½-5.
49	do.	13½	9	31	5	27	4	32	7	6¾-4¾-4-3¼.
50	do.	13½	10½	31	0	27	1	32	2	6½-4⅞-3⅝-3-2¾,
51	do.	13	9½	31	0	27	0	32	4	14½-9½-6-4-3½.
52	do.	12	9¼	31	0	30	7	35	7	3⅞-4-4-4½-4½-5-5¼-5¼.
53	do.	13½	10	31	9	29	0	35	10	5½–4¾–5¼–(crushed–sawed off 2′ 9″)-3⅝.
54	do.	15½	12	31	0	26	10	31	7	7-5⅛-3⅞-3⅛-2⅞.
55	do.	12¼	9¼	30	6	29	11	35	0	September 18th: 4¼-4¼-5½-5¾ sawed off 6½-Sept. 20th, 2.

56	do.	12¼	9¼	31	0	28	0	32	1	10-5¾-4½-4-4-4¼.
57	Hemlock.	14	10¾	28	8	26	10	34	2	6⅞-7-5¾-4¼.
58	Spruce.	14½	9½	32	0	27	3	31	3	10¾-4-3¼-2½-2¾.
59	do.	12¼	9	31	2	29	11	34	7	5¾-4½-4-3¾-3½.
60	do.	14	10	30	9	29	1	34	3	4⅞-3⅝-3⅝-3¼.
61	Hemlock.	13	10	30	8	27	11	33	3	7⅛-5¾-4⅝.
62	Spruce.	13½	10½	30	7	28	11	34	1	4¼-4-3⅞-3⅜-3.
63	Hemlock.	16½	12	31	3	25	9	31	0	7½-7¼-6-5¼.
64	Spruce.	12¾	9¾	31	0	28	0	32	10	4½-4¼-4-4-4-4.
65	Hemlock.	12½	10	23	7	23	2	35	7	16-18¾-19½.
66	Ash.	12	8¾	29	0	27	4	34	3	5¼-4½-3⅞.
67	Spruce.	12¾	9¼	31	0	28	1	33	0	6-5½-5-5-5.
68	Hemlock.	12	8¾	30	4	27	0	32	7	7-5½-4½-4½.
69	do.	13¾	12	30	4	26	3	32	1	10½-10¾-10½-10.
70	Spruce.	15	11¾	32	0	29	0	33	1	4-3½-3¾-2¼.
71	do.	14¼	11¼	31	0	28	3	33	4	5⅝-4½-3¾-3.
72	Ash.	13½	9½	30	9	27	10	33	2	5¾-4¼-3⅜.
73	Spruce.	12¼	9¾	30	7	27	10	33	4	11½-7½-5½-5-3½.
74	do.	12¼	10¾	32	0	27	4	31	5	5½-4¼-3¼-3¼.
75	do.	14	10½	32	0	26	11.	31	2	5¾-4¼-3½-2¾.
76	do.	12	8½	27	7	27	3	35	10	14¾-9¼-6-5-4.
77	do.	12	8¾	31	0	28	3	33	4	4¾-4½-4-4-4.

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